Quadrotors UAVs Swarming Control Under Leader-Followers Formation

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Abstract— **Unmanned Aerial Vehicles (UAVs) swarming has took an important part of the recent researches. Multiple robots offer many advantages when comparing to a single one such as reliability, time decreasing and multiple simultaneous interventions. This paper proposes a new scheme for trajectory tracking of multiple quadrotors UAVs under a centralized leader-followers formation strategy. Attitude stability and position control are assured using a double loop control structure based on the Linear Quadratic Regulator (LQR), moreover the leader-followers formation is maintained via a Sliding Mode Controller (SMC) controller. Many scenarios are proposed within this work. Simulation results proof the energy optimization and formation controller's robustness and accuracy.**

Keywords—quadrotors; swarm; formation; LQR; SMC; trajectory tracking; quaternions.

I. INTRODUCTION

Quadrotors are a type of UAVs named the Vertical Take-off Landing (VTOL) systems. They are capable of hover, forward flight and vertical takeoff landing, as a drawback, the nonlinear, under-actuated dynamic system and the high energy consumption can be mentioned.

Due to the gimball lock phenomena caused by the Euler Angles modeling approach [4,7,8], many works in the quadrotors control literatures such as [3,6,9,10] use quaternions to simplify the modeling algebra. A comparison of the different linear and non-linear controller using the quaternion approach can be found in [3].

However the previous works was able to stabilize the quadrotors attitude. Others such as [1,5,6] focus on the trajectory generation and path following using the differential flatness approach. This approach is often used because it optimizes the output spaces and gives them a direct relation with the states and their derivatives, which then simplifies the trajectory generation problem.

In the other hand it is clear that the drone swarming becomes widely used nowadays, this is due to the advantages offered by the swarm like the reliability, mission time decreasing and multiple simultaneous interventions when executing a task. In [10] the theory of multiple UAVs formation and control is describes. Many control mechanisms and several controllers can be used to hold the formation shape, this is done by controlling the position of the UAVs such as [2], or by the angular and linear speeds control like in [4].

This paper introduces a new concept of the quadrotors swarming formation modeling and control that overcomes the drawbacks and maximizes the performances.

The formation control scheme is designed only with an attitude controller for the swarm agents, which then allows the formation to be maintained with a minimum of a sharing data, and makes the controller more robust to any external disturbances.

Furthermore both of trajectory tracking and the formation control algorithms are based on a double loop control structure with an LQR and SMC controllers. This architecture minimizes the energy consumption and secures the swarm from any collisions between the different UAVs.

This article is organized as follow: Section II discusses the quaternion algebra and the quadrotors dynamic model using a Newton-quaternions approach. The linear model of the quadrotors dynamics is than derived in order to be later used. Section III introduces the double loop attitude and position control using the LQR controller, while Section IV show the formation control design using the SMC control strategy. Section V gives the simulation results with many proposed scenarios. Finally in Section VI future recommendations and the conclusion of this work are given.

II. QUADROTORS MODELING

Quadrotors is a VTOL flying robot includes a rigid cross frame with four rotors. It is considered as a non-linear, unstable, under-actuated system with 4 control inputs (the engine speed rotation) and 6 degrees of freedom (3 translations and 3 rotations).

Two main approaches are used to model a quadrotors, the Newton-Euler formulation and an Euler-Lagrange energy based approach. For both methods a problem of singularity emerges whenever $\theta = \pm\frac{\pi}{2}$ due to the gimball lock phenomena. To avoid this problem an alternate method of the Euler angles in order to describe the body orientation is using the quaternions.

Unit quaternion representations can enable the motion control systems to be all-attitude capable. Furthermore, the

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quaternion-based attitude propagation algorithms are globally nonsingular and numerically robust.

As illustrated in Fig.1. let $E_i\{x_i, y_i, z_i\}$ denotes the inertial frame fixed with the earth while $E_b\{x_b, y_b, z_b\}$ denotes the body frame attached to the quadrotors body.

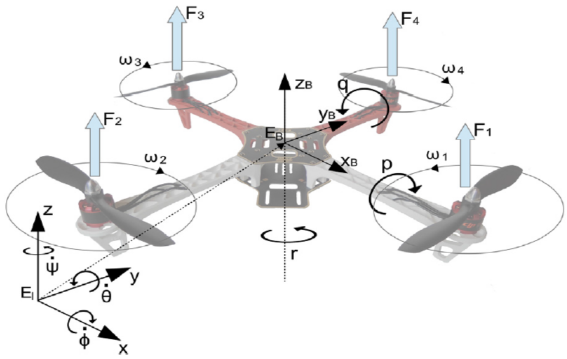

Fig. 1. Inertial and body-fixed frame of the quadrotors

The dynamical model representing the quadrotors rotations can be given as follow:

$$\dot{R_q}(t) = R_q(t)S\big(\omega_b(t)\big)$$

$$J\dot{\omega}_b(t) = -S\big(\omega_b(t)\big)J\omega_b(t) + \tau(t) \quad (1)$$

Where $\omega_b(t) = [\omega_{b1}(t), \omega_{b2}(t), \omega_{b3}(t)]^T$ is the angular velocity in the body frame E_b, J is the inertia matrix and $\tau(t) = \tau_u(t) + \tau_{ext}(t)$, where $\tau_u(t) = [\tau_{u_x}(t), \tau_{u_x}(t), \tau_{u_x}(t)]^T$ are the input torques and $\tau_{ext}(t)$ are the external torques applied on the quadrotors in the body-frame. $R_q(t) \in SO(3)$ the rotational matrix mapping vectors expressed in the body frame into the vectors expressed in the inertial frame E_i, and:

$$S\big(\omega_b(t)\big) = \begin{bmatrix} 0 & -\omega_{b3}(t) & \omega_{b2}(t) \\ \omega_{b3}(t) & 0 & -\omega_{b1}(t) \\ -\omega_{b2}(t) & \omega_{b1}(t) & 0 \end{bmatrix} \quad (2)$$

Lets the unit quaternion $q(t) \in \mathbb{H}$; $\bar{q}(t) \in \mathbb{R}^3$; $q_0(t) \in \mathbb{R}$ is defined with:

$$q(t) = q_0(t) + \bar{q}(t) = q_0(t) + [q_1(t), q_2(t), q_3(t)]^T.$$

With $q(t)$ is a given quaternion, $\bar{q}(t)$ is the complex and $q_0(t)$ is the scalar parts of $q(t)$. The unit quaternion should satisfy:

$$q_0^2(t) + \|\bar{q}(t)\|^2 = 1 \quad (3)$$

.

The rotation of any quaternion vector is given using the three-dimensional matrix with:

$$R_q = (q_0 I + q_{13\times})^2 + \bar{q}\bar{q}^T$$

$$= (q_0^2 - \bar{q}^T\bar{q})\, I + 2q_0 q_{13\times} + 2\bar{q}\bar{q}^T \quad (4)$$

The quaternion derivative is derived using the multiplication of the quaternion q and the angular velocity of the system $\omega(t)$, as follow:

$$\dot{q}(t) = \frac{1}{2} q(t) \otimes \omega_b(t) \quad (5)$$

Furthermore, The Euler angles can be converted into quaternions using the following equation:

$$q_d = \begin{bmatrix} \cos\frac{\varphi_d}{2}\cos\frac{\theta_d}{2} \\ \sin\frac{\varphi_d}{2}\cos\frac{\theta_d}{2} \\ \cos\frac{\varphi_d}{2}\sin\frac{\theta_d}{2} \\ -\sin\frac{\varphi_d}{2}\sin\frac{\theta_d}{2} \end{bmatrix} \quad (6)$$

Finally the dynamic model of a quadrotors using Newton-Quaternion equations is described as follows:

$$\ddot{p} = q \otimes \frac{T}{m} \otimes q^* + \bar{g} \quad (7)$$

$$\dot{q} = \frac{1}{2} q \otimes \omega \quad (8)$$

$$\dot{\omega}_b = J^{-1}(\tau - \omega_b \times J\omega_b) \quad (9)$$

Where $p \in \mathbb{R}^3$ is the position and $\dot{p} \in \mathbb{R}^3$ is the velocity vectors with respect to the inertial frame, m is the quadrotors mass, $\bar{g}$ represents gravity vector, and T defines the thrust vector generated by the quadrotors motors.

The thrusts and reactive torques produced by each rotor are depending on the thrust constant k_T and the drag constant k_D and can be given by:

$$T_i = c_T \rho \pi r^4 \omega_i^2 = k_T \omega_i^2$$

$$\tau_i = c_D \rho \pi r^5 \omega_i^2 + J_r \dot{\omega}_i = k_D \omega_i^2, i = 1,2,3,4 \quad (10)$$

With ρ is the air density, r the radius of the propeller, c_T and c_D are the thrust and drag coefficient respectively that depend on blade rotor characteristics, such as number of blades, chord length of the blade, and cube of the rotor blade radius , and J_r is the moment of inertia of the rotor.

The designed control inputs are given by Equ.(11) :

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} k_T(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ lk_T(\omega_1^2 - \omega_2^2 - \omega_3^2 + \omega_4^2) \\ lk_T(\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2) \\ k_D(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2) \end{bmatrix} \quad (11)$$

Where l is the distance from the motor axis of action to the center of mass.

In order to make the application of a linear controller to the quadrotors possible, the system dynamic equations 7-9 is then linearized around an hovering point where: $\dot{p} = [0,0,0]^T$, $\omega = [0,0,0]^T$ and $q = [1,0,0,0]^T$, thus:

$$\ddot{p} = \begin{bmatrix} g(2q_0q_1) \\ -g(2q_0q_2) \\ \frac{T}{m} - g \end{bmatrix} \quad (12)$$

$$J\dot{\omega} = \tau \quad (13)$$

$$\dot{q} = \frac{1}{2}\begin{bmatrix} 0 \\ \omega \end{bmatrix} \quad (14)$$

IV. QUADROTORS CONTROL

The double loop control structure used in this work is shown in Fig.2. The trajectory tracking controller consists of two parts, an inner attitude controller and an outer position controller loop. The position controller generates the desired position quaternion value q_d and the desired trust T_d for the attitude controller, while the outputs of the attitude controller are the desired angular speeds. It is also important to note that the position quaternion is having the 4th element equal to zero.

Lets $x_A = [\bar{q}\ \ \omega]$ and $x_P = [p\ \ \dot{p}]$ the attitude and the position state vectors respectively. The state space model of the quadrotors dynamics is given as follow:

$$\dot{x_A} = \begin{bmatrix} 0_{3\times3} & 0.5I_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix} x_A + \begin{bmatrix} 0_{3\times3} \\ I_q^{-1} \end{bmatrix} \begin{bmatrix} \tau_{u_x} \\ \tau_{u_y} \\ \tau_{u_z} \end{bmatrix} \quad (15)$$

$$\dot{x_P} = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix} x_P + \begin{bmatrix} & 0_{3\times3} & \\ g & 0 & 0 \\ 0 & -g & 0 \\ 0 & 0 & \frac{1}{m} \end{bmatrix} \begin{bmatrix} u_{xd} \\ u_{yd} \\ T_d \end{bmatrix} \quad (16)$$

In order to assure the system stability and find the optimum solution for a problem of minimization, an LQR controller is used for both attitude and position loops.

The Equ.(17). represents the quadratic cost function to minimize:

$$J(x,u) = \frac{1}{2}\int_0^{\infty} (x^T(t)Q_x(t) + u^TRu(t))dt \quad (17)$$

With R and Q are weight matrix used to increase or to diminish the effect of the system states. Both matrices are selected by the designer depending on the required performance.

The optimum input is defined as U = -Kx With $K = R^{-1}B^TP$ and P is the solution to the Ricatti differential equation given by:

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \quad (18)$$

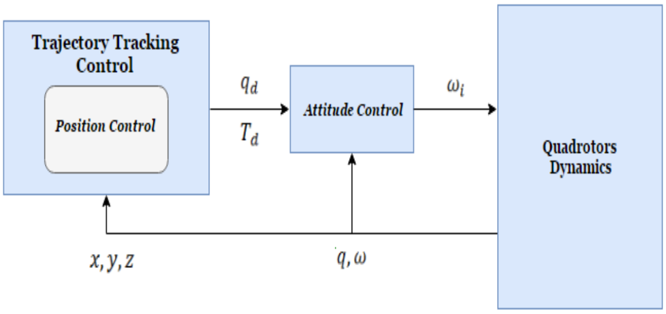

Fig. 2. Block diagram of the proposed control structure

V. FORMATION CONTROL

In this section a control formation scheme is proposed in order to maintain the centralized Leader-followers formation. As shown in Fig.3. in a formation control the UAVs are not physically coupled, but they are strongly constrained to keep a pre-defined distance between them, and then the desired formation.

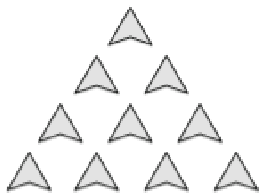

Fig. 3. Leader-Follower Formation

Considering a team of I quadrotors under a leader-follower approach, such that $i \in \{L, 1,2,3, \dots, N\}$, where L is the leader while N are the number of followers.

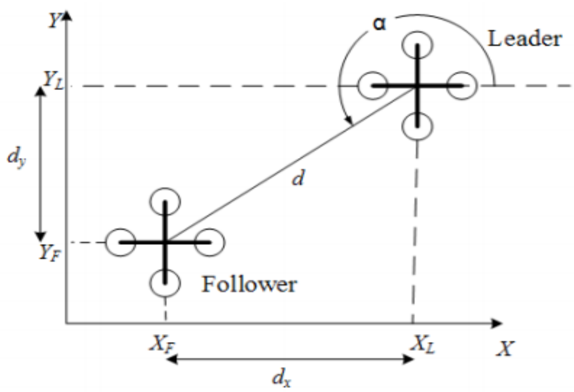

Fig. 4. Leader-Followers Formation architecture

In a formation control the objective of the leader-follower formation controller is to achieve the desired configuration in X-Y plane. The agents can be at the same desired altitude Z or at different altitudes. The formation topology is maintained via keeping a constant distance d and a desired angle α between each agent and the leader of the swarm.

$$d_x = -(X_L - X_F)\cos(\psi_L) - (Y_L - Y_F)\sin(\psi_L)$$
$$d_y = (X_L - X_F)\sin(\psi_L) - (Y_L - Y_F)\cos(\psi_L) \quad (19)$$

with d_x and d_y are the X and Y coordinates of the actual distance d as in Fig.4.

The proposed control algorithm is shown in Fig.5. by applying the SMC controller we aim to keep the formation even in perturbed or uncertain environment. Equ.20. presents the conditions to be satisfied for the formation control errors of x; y and z should:

$$\lim_{t\to\infty}\|e_x\| = \|{d_x}^d - d_x\| = 0$$
$$\lim_{t\to\infty}\|e_y\| = \|{d_y}^d - d_y\| = 0 \quad (20)$$

Where ${d_x}^d$ and ${d_y}^d$ are the desired distance between the leader and follower in the X and Y coordinates respectively.

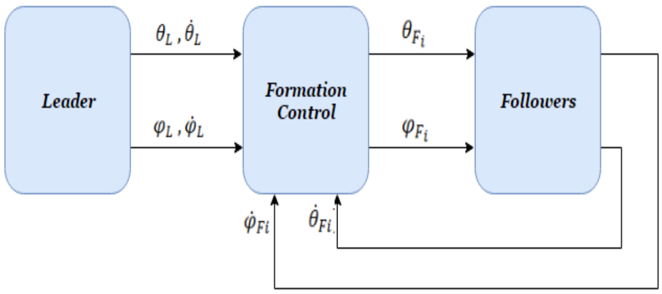

Fig. 5. Leader-Followers Formation control

Lets S(t) be , a time varying surface is defined by the scalar equation s(e; t) = 0. The goal is to design a first-order sliding mode controller to minimize this error. where:

$$s(e,t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e \quad (21)$$

The second-order tracking problem can be transferred to a first-order stabilization problem such in [8] , thus:

$$\dot{s} = \ddot{e} + \lambda\dot{e}$$
$$\frac{1}{2}\frac{d}{dt}s^2 \leq -\eta|s| \quad (22)$$

Equ.(22) is a Lyapunov candidate function chosen for the control law u to maintain scalar s = 0. This function states that s^2 is the squared distance to the sliding surface, where η is a positive constant.

The formation can be then controlled for each follower as in [2] using the following equations:

$$\ddot{X}_{F_i} = \ddot{X}_L + \lambda_x(\dot{X}_L - \dot{X}_{Fi})$$
$$\ddot{Y}_{F_i} = \ddot{Y}_L + \lambda_y(\dot{Y}_L - \dot{Y}_{Fi}) \quad (23)$$

Finally, the position control problem is transformed to an attitude control, which means that a direct estimation of the attitude can be used to control the swarm:

$$\theta_{F_i} = \theta_L + \lambda_\theta(\dot{\theta}_L - \dot{\theta}_{Fi})$$
$$\varphi_{F_i} = \varphi_L + \lambda_\varphi(\dot{\varphi}_L - \dot{\varphi}_{Fi}) \quad (24)$$

where λ_θ and $\lambda_\varphi$are the formation control gains, with $\lambda_\theta > 0$ and $\lambda_\varphi > 0$.

VI. Simulations Results

In this section the simulation results related to the quadrotors formation discussed in the other sections is shown.

Table.1 presents all the parameters adopted to the quadrotors model used in the simulation.

TABLE I. QUADROTORS PARAMETERS

Parameter	Value	Unit
I_x	0.00065	$kg.m^2$
I_y	0.00065	$kg.m^2$
I_z	0.0014	$kg.m^2$
l	0.125	m
k_T	0.001	$kg.m$
k_D	0.00002	$kg.m^2$
M	0.26	Kg

Once compared with the estimated Euler angles from the dynamic model, the attitude signals are then converted into PWM radio signals and scaled in a magnitude rang Attitude commands are interpreted from the input PWM radio signal and scaled to range in magnitude from -1 to 1.

In order to make the designed controller suitable for a real implementation, a simulation rate of 200 Hz was chosen. The necessary constraints and limitations over the dynamic model and energy consumption were all taking into consideration.

Three scenarios with a three UAVs under a centralized triangular leader-followers formation (Fig .6.) have carried out as follow:

1- *Scenario1:* All the 3 UAVs start from the same point and track a rectangular path.
2- *Scenario2:* In this case the UAVs are tracking the same path as the Scenario 1 but in the presence of an external wind disturbance on the leader and followers.

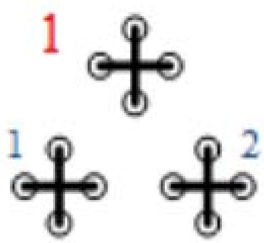

Fig. 6. Leader-Followers Triangular Formation

A- Scenario 1 :

As mentioned before, in this case the 3 quadrotors UAVs are sicked to start from the same departure point $P_0(x,y,z) = (0,0,0)$ and follow a rectangular path of (1m * 1m) at a 1 m of altitude and maintain their formation, the distance d to keep between the UAVs is choosen as: d = 10m.

This first scienario was chossen in order to test the controller and its ability to hold the swarm formation while tracking a desired path.

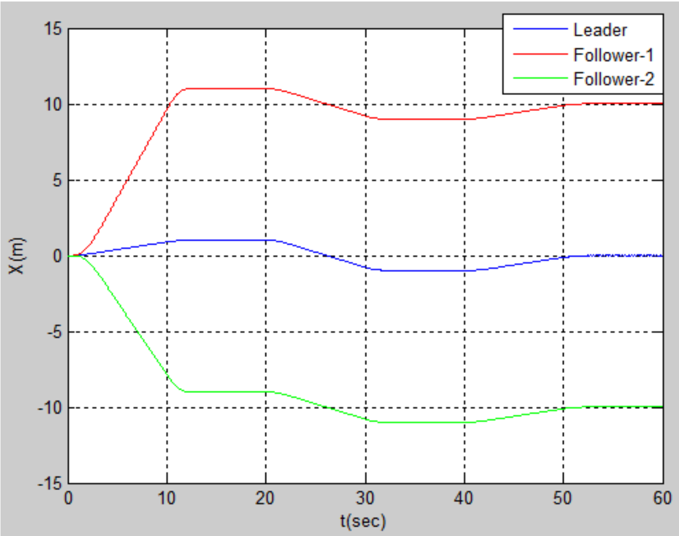

Fig. 7. Swarm trajectory along the x-axis

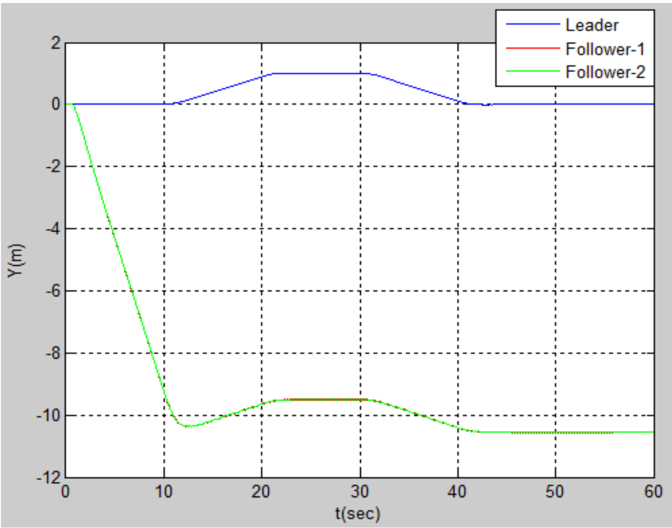

Fig. 8. Swarm trajectory along the y-axis

Fig. 7,8,9 show the obtained result for the trajectory in the three coordinate X,Y and Z. It is clear that using the LQR controller all the quadrotors UAVs was able to track the desired path with a high accuracy. In othor hand the UAV followers succeed to follow their leader by keeping the separation distance (10 m) and maintain the swarm formation.

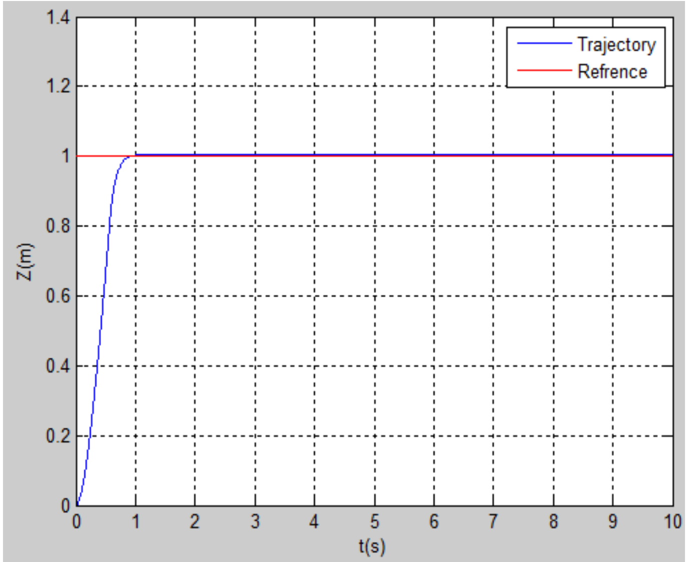

Fig. 9. Swarm altitude refrence

For the altitude hold, the response was the same since all the UAVs are sharing the same altitude.

Fig.10. shows the obtained unit quaternion used for the attitude stability, where it is clear that the quadrotors attitude stability is assured, as designed before, around the pre-defined unit quaternion $q = [1,0,0,0]^T$.

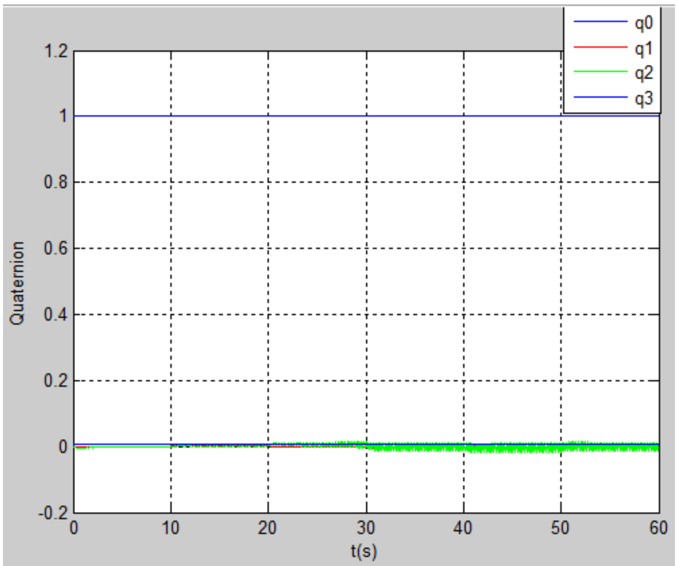

Fig. 10. PWM input control signals

The input PMW control signals generated by the four rotors are depicted in Fig.11. It can be noticed that all the signals of the control inputs are within the ESC (Electronic Speed Controller) functional range of 1-2 sec which correspond to the real case. The obtained signals represent the energy optimization of the controller. The high oscillations are due to the altitude hold but they are not considered as very high frequency oscillations that affect the engines health.

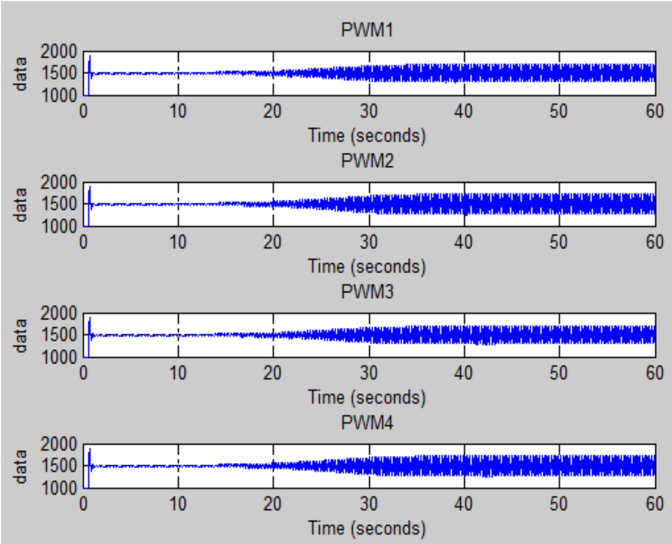

Fig. 11. PWM input control signals

B- Scenario 2:

In this scenario the UAVs swarm is tracking the same path as in scenario 1 but this time by facing an external wind disturbance simulated as a band-limited white noise of 0.1 PSD (Power Spectral Density) starts from the 12th second, The perturbation is added to the theta angel over the X-axis. This kind of scenarios allows testing the robustness of the designed controller.

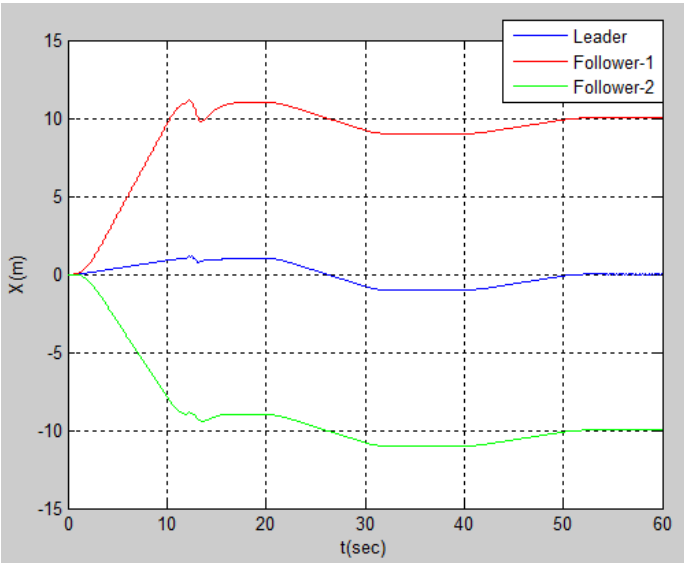

Fig. 12. Swarm trajectory along the x-axis in the presence of a perturbation

From Fig.12. the robustness of the controller is proofed as the disturbance has not influence the swarm formation, and the UAVs kept their stability after just 2 sec in the leader case, and 1 sec for the followers due to the SMC formation controller.

IV. CONCLUSION

In this paper the problem of the quadrotors swarming formation control was studied using a new scheme to hold a centralized leader-followers formation. Many scenarios were proposed in order to test the performances and the robustness of the designed controllers, all the obtained results were judged to be satisfactory where the quadrotors UAVs have successfully tracked the generated path and eliminate the effects of the external wind disturbances.

The combination of the double loop LQR controller for path tracking, and an SMC controller for the formation control, was applied to maintain the swarm formation strategy.

The next step for this research will be the implementation of those algorithms in a real quadrotors swarm and add the obstacle avoidance ability to prevent collisions, then apply this strategy in some real applications such as surveillance, fire extinguishing and search & rescue (SAR) operations.

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